Problem solving: logic exercise

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	1	2	3	4	5
Nation					
Colour of house					
Drink					
Animal					
Food					

1. The English person lives in the red house.
2. The Spaniard owns a dog.
3. Coffee is drunk in the green house.
4. The Ukranian drinks tea.
5. The green house is immediately to the right of the ivory house.
6. The person who eats pork owns snails.
7. Steak is eaten in the yellow house.
8. Milk is drunk in the middle house.
9. The Norwegian lives in the first house.
10. The person who eats mutton lives next door to the person with the fox.
11. Steak is eaten in the house next to the house with the horse.
12. The person who eats fish drinks orange juice.
13. The Japanese person eats vegetables.
14. The Norwegian lives next door to the blue house.
15. The coffee drinker has a tortoise.
16. The Ukranian's neighbour drinks water.

Two books - *Communication Games: Participant's Manual* by Karen R. Krupar (The Free Press, New York, 1973) and *Common Ground: A Course in Communication* by R.K. Sadler and K. Tucker (Macmillan, Australia, 1981) - contain versions of the problem printed here. When we found, in the 1970s, that craft and technician engineering students at the then Tottenham College (now the College of North East London) enjoyed working on this problem, a colleague and I made up six others in the same format and used them often over the period till 1990. I put one of these in my article 'Teaching Logical Problem Solving' in *General Educator* 13, Nov/Dec 1991.

We used to give each student a copy of the grid and statements, and then explain that there was only one correct solution, and that, as the problems were purely logical, they must put aside any knowledge of actual nationalities, foods etc. They would then work individually on the problem, and we would give help when asked. Afterwards we might go through the problem on the board, inviting them to challenge us if they thought we had asserted something without proving it. However, a whole class could work together on such a problem if each student were given a sheet containing only the clues, with the grid being done on the board. Another possibility would be to give each student only one or two clues, which he or she must then share through speaking.

The discussion below focuses on the logical side of this problem and does not take into account psychological and cultural issues that arise from it. It also ignores the relation between problems like this and real life ones, though clearly the attempt to identify techniques learnt through solving this problem may suggest real life uses for those techniques. However, it is worth pointing out that although this problem differs from real life ones by being more static, less messy, less important, less subject to time pressures and so on, it can nevertheless stimulate the intellect of the person who tries to solve it to a high level of activity and often of confusion. The main value of such work is that it enables teachers and students, both individually and collectively, to think about how they think now and how they might think in future.

Because they are used to starting written work at the top left of the page, some students may start by writing 'English' and 'red' in the first house. Most, however, will understand from the outset that the first item to be fixed could be anywhere on the grid, and this will lead them to divide the statements into those which tell you where an item goes on the grid and those which tell you the relationship between two or more items without telling you where they go. Searching through the statements on this basis, most students will then write 'Norwegian' in the first house and 'milk' in the third house. And most, again, will then work out that the second house must be blue. (This is often the first sign that students are starting to recognise what kind of reasoning is required.)

From this point, students usually move forward either by stumbling on the correct route and following it, or by following the wrong route without knowing they had a choice, or by taking a 50/50 chance that turns out to be right, or by taking a 50/50 chance that turns out to be wrong, or by asking for help. (The first four approaches will be considered later.)

If a student asks for help at this stage, his or her attention is probably focused on statement 5. The possibilities arising from this statement are as follows:

- the ivory house and the green house must be next to one another, that way round;
- the first house cannot be green because there is no place for ivory to the left of it;
- the first house cannot be ivory because the second house is already known to be blue, not green
- because the second house is blue not ivory, the third house cannot be green;
- the fourth house could be green - in which case the third house must be ivory;
- alternatively, the fifth house could be green - making the fourth house ivory.

If the student has already worked out these points, or has worked out most of them and can understand the others when you explain them, one way to give help is to prompt him or her to consider another statement in conjunction with 5, namely statement 1. Clearly 'English' and 'red' cannot go into the first house, because we already know that the person living there is Norwegian. And they can't go into the second house because we already know that house is blue. Finally, they can't go in the fourth house, because they would then prevent ivory and green from being next to one another. So 'English' and 'red' can only go either in the third house or in the fifth house.

Question

So we seem to be back again with the problem that led the student to ask for help in the first place - except that the question now is whether the third, fourth and fifth houses are red / ivory / green or ivory / green / red. But in fact we can now fix a fourth item in the grid, because, whichever of these two arrangements for the last three houses is right, the first house must be the one remaining unused colour, yellow. This then allows us to fix two further items, the steak and the horse (statements 7 and 11).

There is also another step we can take without deciding about the last three colours. We know that the Ukranian him/herself drinks tea (statement 4) and that he or she is next door to someone who drinks water (statement 16). About these items we can now reason as follows. The Ukranian cannot be in the first house, because we already know that that's where the Norwegian is (therefore the water cannot go in the second house). The Ukranian and the tea could be in the second house, in which case the drink in the first house must be water. Is this the only possibility? If so, why?

Obviously the Ukranian can't go in the third house because he or she drinks tea not milk. But he or she also can't go in the fourth or fifth houses, because one of these houses must be green, and we know from statement 3 that the drink in the green house is coffee. Given that the drink in the middle house is known to be milk, then, the fact that either the fourth or the fifth house must drink coffee, means that the combination of Ukranian / tea / water cannot fit into the last two houses. So the only possibility is that the Ukranian is in the second house.

By this route, statements 4, 7, 8, 9, 11, 14 and 16 will now have been used. With one exception (statement 10), each of the remaining unused statements links two items within one or other single house: English with red (statement 1); Spaniard with dog (statement 2); green, coffee and tortoise (statements 3 and 15); snails and pork (statement 6); orange juice and fish (statement 12); and Japanese and vegetables (statement 14). We can show this by drawing a number of separate vertical columns (ie 'houses') on another piece of paper, and then going through the unused statements gathering up this loose information. (It turns out that we need six of these columns.)

None of these columns can be fitted into the first or second houses on the grid itself. On the other hand, three of them could be fitted into the third house. Therefore we know two further things: that we still can't say whether the colour of the third house is red or ivory; and that sooner or later we shall need to reduce these six freestanding 'houses' to three, corresponding to the third, fourth and fifth houses on the grid. Which of these six freestanding 'houses', then, can we fit into one another (thus allowing the overall number of them to be reduced) and which - if any - can we not? We notice the following points:

- the 'house' containing English and red can be merged either with the 'house' containing snails and pork, or with the 'house' containing orange juice and fish;
- the 'house' containing Spaniard and dog can only be merged with the 'house' containing orange juice and fish;
- merging English and red with orange juice and fish, then, would leave no place for Spaniard and dog;
- the 'house' containing green, coffee and tortoise can be merged with the 'house' containing Japanese and vegetables;
- no other merger of columns off the grid is possible.

Therefore we now have three 'houses' still off the grid:

- a 'house' containing English, red, snails and pork;
- a 'house' containing Spaniard, orange juice, dog and fish;
- a 'house' containing Japanese, green, coffee, tortoise, vegetables.

If we now try to fit these three 'houses' onto the grid, we see that only one of them, the first, is compatible with what we already know about the third house (ie because the other two both have drinks in them already). Therefore we now know that the middle house is red, and we can complete the grid, not forgetting statement 10.

The solving procedure set out here does not involve taking a chance at any point (except possibly in your head or in rough as you decide what next to put on the grid). Therefore if students are to complete the grid without ever writing anything on it which goes beyond what they can prove, they must proceed more or less along this route, with only minor variations. However unless students ask for help, they don't usually solve the problem in this way. Therefore we need to look at the other approaches they tend to use.

Some students will see that ivory / green could be in the fourth and fifth houses, will put them there and simply carry on till they complete the grid - that is, without being aware that this was only one of two possibilities. Others will see that ivory / green could be in the third and fourth houses and proceed on that basis - that is, without seeing that there was another possibility - until they find out that they've gone wrong.. Others will see that there are two possible positions for these colours and try the position that is in fact correct, thereby eventually completing the grid. That is, they are aware that there were two possibilities and hence aware that they have proceeded by trial and error at this crucial point. Finally, others still will see that there are two possibilities and try out first the one that is not correct.

For this last group of students, and still more for those who base themselves on the wrong position of ivory / green without knowing they had a choice, the crucial thing is what happens when they later reach a poiny where it's obvious they've gone wrong. Will they be able to find their way back to the point where they went wrong and follow the other route or not? And if they do, will they realise straightaway that this other route must be right, or will they think that they are taking a chance in following it? To see what is at stake here we need first to identify more clearly the reasoning techniques that students may learn from this exercise.

The techniques involved include at least the following:

- distinguishing in practice between relationships and positions;
- operating on relationships without knowing positions;
- combining information about positions with information about relations so as to determine further positions;
- combining two pieces of given information to infer a third that is not given;
- systematically narrowing down possibilities until only one remains;
- switching between these last two techniques; for example, checking after each extension of the known area within the grid whether or not this has narrowed the unknown area to a point where there is only one possibility for a

given box; (This could also be described as learning that in a closed-ended problem of this type, to add to what you know necessarily reduces what you don't know, and vice versa.)
- constructing models off the grid and working between these and the items so far fixed on the grid;
- organising and re-organising the given information so as to clear that which has already been fully used out of the way;
- determining which groups of related items are compatible with one another and which are not;
- working with groups of items rather than proceeding one box at a time.

All of these techniques are involved in solving the problem without taking a chance. Where a chance is taken, and it is wrong, some or all of these further techniques may then be required:
- scrapping the whole reasoning process up to the point reached and starting again;
- tracing your steps back from the point where you realise you are wrong until you find the point where you went wrong and then proceeding along the other route;
- consciously marking the point where you make a choice so as to be able to go straight back to it if that choice proves eventually to be wrong.

Stumble

Students who stumble on the correct path through this problem without seeing that there are other possibilities learn less from it than students who do it in any of the other ways mentioned earlier. One way to stop this, although it would also stop the learning that could result from stumbling on to the wrong path, is to tell students at the start that they must not take any step which they cannot prove to be correct at the time of taking it. Another way to encourage students to be more reflective in their approach to the problem is to ask them to write the number 1 in the first box they fix (ie along with the appropriate word), 2 in the next, and so on up to 25. This enables the sequence they have used to be reviewed, both while they are still working on the problem and afterwards, and it makes it easier to compare the procedures used by different students. (They could also be asked to write in each box the letters indicating the clue or clues used to fix that item.) There are two potential difficulties about doing this. First, we don't want to kill the student's enjoyment and/or his or her focus on the problem itself. Secondly, numbering in 25 steps is not a very good reflection of what actually happens, which more often involves seeing several things all at once, then pausing for reflection, and then seeing another cluster of items and positions.

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