Problem solving: does Vygotsky's model fit logic problems?

Colin Waugh

This article considers how far the writer's main experience of problem solving with FE students relates to the model of concept development proposed in the early 1930s by the Russian psychologist Lev Vygotsky.

Reference will be made to the use by the writer and a colleague, with engineering students at a college in north London during the 1970s and 1980s, of one type of logic problem. [For an example, see pp23-24 of this issue. Ed.) In these problems the student is given with a blank grid containing twenty five boxes. Each vertical set of five boxes represents a 'house', and each horizontal row represents the nationality of each house's occupant, the colour each house is painted, the type of food and/or drink consumed, the car driven, the pet kept, or the like. All twenty five items are named in statements about the houses which are also given to the student. The student has to make inferences from this incomplete information about the inter-relation of these items, such that he or she eventually ends up with a completed grid conforming to all the statements. Only one layout is correct. To allow review of the process, the student may also be asked to write the number 1. in the first box which he or she fills in, 2. in the second and so on. Six different versions of this problem were used, five of which were devised by the lecturers concerned. However, two versions that were fairly easy, one devised by us and one acquired from elsewhere, were used more often than harder ones. At least 2,000 students worked on these easier problems in the fifteen year period.

Vygotsky derived his model of concept development from the responses of subjects to an exercise which was devised by himself and his collaborators but which was based on one developed earlier by the psychologist N. Ach. In Vygotsky's version, the subject is presented with a surface on which are spread in a random fashion between twenty and thirty wooden blocks. Features of these blocks include six shapes (rectangle, trapezium, triangle, hexagon, semicircle and circle), five colours, two thicknesses or heights, and two sizes (small and large). No two blocks are the same. Each block has one of four nonsense words written on its underside, such that this word is visible to the subject only when the psychologist conducting the exercise turns the block over. These words are placed as follows: one word on the base of all small and flat blocks, another on all small and think blocks, a third on all large and flat blocks, and a fourth on all large and thick blocks. At the start, the psychologist turns over any block (for example a small, thin, yellow triangle) and asks the subject to pick out other blocks that he or she thinks might have the same word written underneath them as this. The exercise proceeds with the subject trying out ways of grouping the blocks. Each time a grouping is produced, the administrator turns over a block which has been grouped with the sample one but which has a different word written on it from that which is on the original sample - that is, a block which has been 'wrongly' selected. (An alternative procedure would be to turn over a block that has the same word on it as the first sample but has not been selected.) Discussion can also take place between the psychologist and the subject each tim this is done. The exercise ends after th subject groups the blocks in such a wa that there are four groups, each membe of each group having the same wor written on its underside. Vygotsky an his collaborators used this exercise wit about three hundred children. A versio of it was also used with mental patient suffering from schizophrenia, and wit people from nomadic cultures in Centra Asia during anthropological fieldwork.

Vygotsky's exercise was partly a investigation in the field of cognitiv psychology, partly a diagnostic test, an partly a device for one-to-one teachin and learning. The houses problems, o the other hand, are teaching and learnin activities adapted from puzzles involvin logical reasoning. They were used wit groups of students, who were normall asked to work individually on a problem at first, with support from a lecturer, afte which there might be a group discussio of the solution, and the techniques fo reaching it. Students were allowed t help one another.

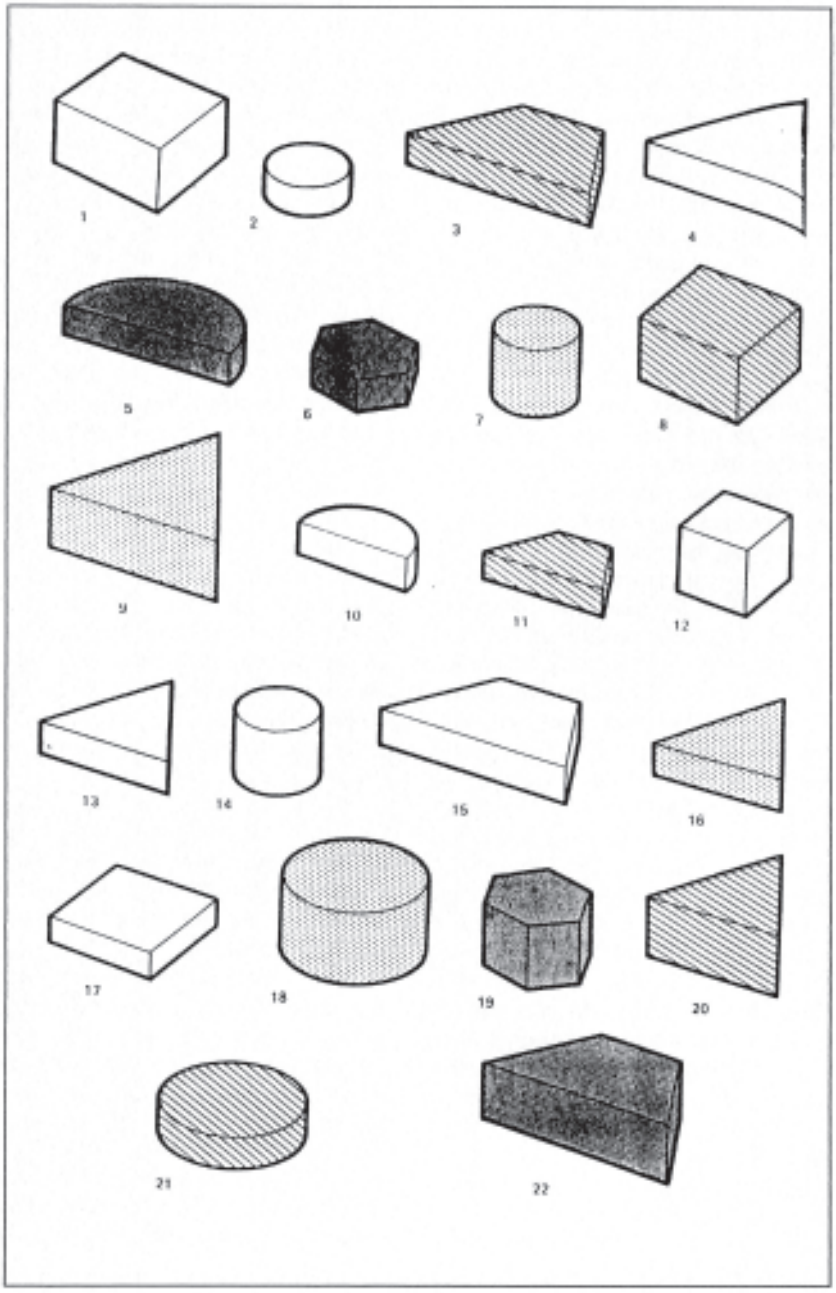
From experience of the blocks exercise, Vygotsky came to think tha subjects' responses could be classed i eleven different phases, which form a sequence from the lowest level o possible response up to a fully correct solution, and which can be grouped int three broader levels - at which the subject forms successively a series o 'congeries' or heaps, a series o 'complexes', and a series of concepts. Any given subject might omit particula phases or even entire levels, or might g through them in a different order, mix them together, or add in some othe approach. Nevertheless Vygotsky

believed that these phases and levels were constant enough to amount to a developmental sequence comparable to those identified by, for instance, Piaget.

The procedure here will be to look at each of the phases in Vygotsky's model which precede or fall short of a correct solution, and also at the transitions between phases and levels, asking in each case whether experience with the houses problems parallels his model.

In Vygotsky's view, the lowest level (1.1) of approach to grouping the blocks is one in which the subject picks blocks at random across the whole spread. In this phase the subject is not acting on the basis of a strategy, however provisional. The equivalent to this in the houses problems could be where a student who meets such a problem for the first time is unable, despite having heard an explanation of the 'rules' of the problem, to see any systematic way of proceeding, and so writes words from the statements at random into the grid as an alternative to doing nothing.

Vygotsky regarded it as an advance on this stage when the subject, after receiving feedback, pushes blocks that are already close to one another more tightly together, or in some other way divides the blocks into two or more groups on the basis of chance features of their initial distribution on the table (1.2). In the houses problems the equivalent to this may be when a student writes at the top of the first column on the grid whatever word is mentioned in the first statement. In both these cases, the person trying to solve the problem starts to adopt a strategy - as opposed, that is, to reacting in a random way - but has not yet recognised the need to free him or herself from the influence of chance factors in the way the problem has been presented. (The statements given with the houses problems are in a random order comparable to the random distribution of blocks on the table.)



In the next phase identified by Vygotsky (1.3), the subject, after further feedback, combines the two earlier approaches. He or she now makes a group by, for example, picking one block at random from each of the groups formed in 1.2. Since the groups formed in 1.2 are determined by random factors, the groupings now produced are objectively no less so. Nevertheless, the subject is now attempting to move towards a systematic approach, in that he or she takes a block from each pre-existing group rather than at random from across the whole field.

The equivalent to this in the houses problems may be when the student starts to work both on the grid - that is, to write words from the statements into boxes - and off the grid - for example, to assemble combinations of items on another piece of paper. A student who, having previously been unable to make progress with the problem, starts to work in this way shows that he or she has realised both that he or she can alter the situation by reasoning, and that the problem itself contains 'tools'- for example lines in the grid or points in the statements - that he or she can use for this. However, only a few students can work through to a solution from this point without help.

At this stage then, the person trying to solve the problem, whether it be the blocks or the houses, sheds the tendency to heap items together, and turns towards approaches which involve planning. However, planning in itself, as distinct from planning based on a combination of analytical with synthetic reasoning, does not guarantee a fully rational approach. The second level identified by Vygotsky includes several strategies, all of which involve planning but fall short of full rationality. He uses the term 'complex' for a grouping of the blocks produced by any such pre-rational strategy.

The first form of complex which Vygotsky identified (2.1) is where the subject assembles a group of blocks each of which has one or more features in common with the sample block. In doing so, however, he or she does not concern him or herself with whether all the blocks selected share the same features, only with the relation between each selected block and the sample. Thus if the sample block is a small, thin, yellow triangle, the group may contain one which is small but not yellow, thin or triangular, one which is thin but not small, yellow or triangular, one which is yellow but not small, thin or trangular, and one which is trangular but not small, yellow or thin. Vygotsky termed arrangements of this type 'family resemblances', because in a family one person may have a common ancestor's nose, another his or her chin and so on, without any individual inheriting all of his or her features.

At this stage, then, the subject has started to think that the words under the blocks must refer to different features of the blocks themselves (as opposed, for example, to where they happen to be on the table) and to consider whether they may refer to features that are common to more than one block. The weakness of this 'family resemblance' strategy is that it is not based on a feature or set of features common to all the blocks grouped together. Thus if, as the subject in effect implies (incorrectly), the blocks grouped in this way all had the same word under them, the meaning of that word could not be distinguished from those of words on the blocks not included in that group.

The equivalent to this in the houses problem could be where the student sees that he or she can form links between items by looking at the statements, but does not yet see that every link must fall into one (and one only) of two categories - those which the statements definitely require and those which they rule out. Thus he or she may form a 'house' (a vertical set of five boxes) and this 'house' might have the right sort of item in each box (for example, a name in the name box, a colour in the colour box) but these items are not necessarily the ones that must logically be combined with the others so placed. Such a student, then, is able to recognise that there are houses and categories of item (for example colours or foods) and that links can be formed amongst them, but he or she has not reached the stage of distinguishing *within* each category between those items which could go together in a 'house' and those which could not.

The second form of complex-type reasoning (3.2) that Vygotsky identified is that which he calls a 'collection'. Here, the subject, having discovered from feedback that the 'family resemblances' approach was wrong, switches over to making groups out of items that contrast with one another and have as few shared features as possible. This is like a person who, in collecting some category of objects (say blues records), tries to include a sample of each type from the field in question (for example, one record by each major acoustic performer, one by each classic female singer, one by each Chicago artist etc). Here, then, the blocks are arranged so that the differences between those in any one group cause them to complement one another. That group as a whole is therefore like a sample, in which all the important features present in the total array of blocks are represented.

In the houses problem, the parallel to this might be when a student acquires a general impression of the exercise and then tries straightaway to produce a completed grid that looks to casual inspection as if it contains all the items, in an order that could be right. Such a student fails to rank in order of importance the information given in the problem. He or she tends, rather, to assume that it must all be equally important, and so he or she tries to produce a 'balanced' picture where everything gets a look-in but where the rules determining what shall be a valid solution are not applied. Such a grid falls apart as soon as you start to check it against the statements in the problem.

The third 'complex' strategy that Vygotsky noted (3.3) was a 'chain'. Here the subject, having been shown that the 'collection' approach does not work, goes back to looking for common features. He or she also recognises that the family approach was too haphazard, and therefore attempts to correct this by separating out a set of blocks in which the first individual or sub-group is linked by one feature to the sample block and by a different feature to the next member of the separated group. Thus if the sample block was a small, thin, yellow rectangle, the subject might pick out first a small thick, blue triangle, then a thick, large, red circle, then a circle that is green, thin and small, and so on. In such a 'chain' the linking features are really present, but each block in the 'chain' also has other characteristics which are not taken into account. After a block has been selected, one feature of that block is picked out as the basis for selecting the next block, but some other feature could just as well have been picked out instead. This procedure, then, is arbitrary, in that it does not incorporate rational criteria for excluding blocks from a group.

Producing a solution to a houses problem involves assembling a chain of reasoning. The student must find a starting point (that is, one or more statements which allow him or her to fix an item on the grid) and then another statement and item that link to that first one, and so on up, to a point where progress becomes a matter of elimination only. He or she thereby reconstitutes either the actual sequence in which the problem was made up, or a sequence in which it could have been made up. When such a chain is constructed in a way that leads to a correct solution, all the information in each statement is taken into account as the statement is put into the chain. However, sometimes the student, while trying to assemble a chain, leaves out of consideration part of the information in a statement. For example in the statement 'the green house is immediately to the right of the ivory house', he or she may overlook the implication of 'immediately'. Even if this happens only once it will take the student in a wrong direction. This tendency to focus on part of the information in a statement and ignore the rest may be like what happens in chain reasoning as identified by Vygotsky.

In the fourth complex type of reasoning to which Vygotsky refers (2.4), the subject composes a group from blocks which he or she imagines to share a common feature. (This comes out in discussion at the point where the psychologist provides feedback.) One example Vygotsky gives is where the subject groups trapeziums and triangles together on the grounds that a trapezium is a triangle with the top cut off. This phase represents an advance over the chain strategy because it shows that the subject is beginning to look for one or more features that are common to some blocks whilst not being shared by others, but it is limited by the fact that these features are not objectively present.

In the houses problem an equivalent to this may be where a student attempts to work within the 'rules' of the problem as explained at the outset by the lecturer, one of which is that real life relations (for example a given nationality's diet) must be disregarded, gets stuck, and then tries to bring some real life relations into the problem to help him or herself over the difficulty. But the relation between the statements and the solution is such that none of these real life factors is objectively present within the problem. There is never a link between the real habits of people and the relations between items in the problem. Therefore in the 'world' of the problem, which has been constructed in such a way that logical considerations outweigh everyday experience, these relations as imported by the student are 'imaginary'.

Vygotsky's last (2.5) phase of complex reasoning is where the subject composes a group from blocks which do all share with the sample block a common feature which is objectively present, but then reveals in discussion that he or she does not understand the logic behind this procedure. For example, the subject may put all the circles, or all the green objects together. On being shown that this is not the solution, he or she does not say 'Oh, I see; so it can't be the shape, colour etc . . .', but instead gives reasons why the block shown to be wrong does not, after all, share the common feature - reasons which can only be 'imaginary'.

Vygotsky terms this the 'pseudo-concept' phase. In his view, the subject's thinking here is like that of a person who can talk knowledgeably about some activity (say, running a marathon) but whose knowledge of this activity comes from talking to people who engage in it rather than from experiencing it him or herself. The point is not that such knowledge is worthless or misleading. On the contrary, Vygotsky believed that language is a tool which humans produce so that they can develop ideas with some degree of independence from experience. Nevertheless, the acquisition of fully rational concepts requires the interaction of language and experience.

In a houses problem, the equivalent to this pseudo-concept phase may be when a student realises, either from the lecturer's explanations or from watching others, what kind of procedures he or she should use for tackling it, but cannot work independently for long because his or her experience of the problem has not been enough to make him or her see why these procedures are necessary within it.

The boundary between complexes and concepts is crossed, in Vygotsky's view, when the subject forms groups from blocks that are as similar as possible to one another (3.1). The key feature of such a grouping is that in forming it the subject necessarily tries to exclude blocks which differ sharply from the most recently displayed sample block and from any blocks already grouped with it in the current 'round'. He or she is thereby using a different kind of approach from those employed previously, all of which were flawed by the fact that logically each of the groups produced could have been extended to include all the blocks.

The weakness of this 'maximal similarity' approach is that it does not draw on all the information that is potentially available. If the subject were to utilise fully the feedback that he or she will have received by this stage (or his or her prior knowledge if he or she goes straight to it), he or she must know that grouping by maximal similarity cannot provide the solution. He or she must therefore be setting aside aspects of that feedback or prior knowledge, either by forgetting it, losing track of it in some other way, or deciding to de-emphasise it.

In the houses exercise the equivalent to that could be when a student who has advanced some way into the problem, and who has taken within it a direction which leads away from the correct solution, declines to accept feedback. The original version of the problem contains a point at which this is particularly likely to happen. At this point, the student is presented with two options for placing a particular item. Some students may be unable to progress beyond this point. For those who do go on, either on their own or with help, there are several possibilities. One possibility is that the student will not pick either of the options but will work out instead a much longer route through the problem, such that he or she avoids ever taking a chance. A second possibility is that the student will pick by chance the option which leads to a correct solution and will just carry on through to the end. A student who does this may or may not be aware that he or she has made a choice. A third possibility is that the student will pick the 'wrong' option, such that after some more moves he or she will find it impossible to continue along that route. Some students for whom this is the case are aware that they have taken a chance between two options, and amongst these some are able to work their way back to that point and take the other option. Others, however, are not aware that they made such a choice, and therefore cannot retrieve the situation except by starting the whole problem again. These students have arguably employed a strategy which falls short of the fully rational in a way that is analogous to Vygotsky's 3.1., in that they have opted not to draw on all the information that was potentially available to them through feedback.

For Vygotsky, a step beyond the 'maximal similarity' strategy occurs (3.2) when the subject, perceiving from feedback that this strategy is not correct, switches over to grouping the blocks on the basis of a single shared feature. Thus he or she now groups all the blocks by shape, colour, size, or height / thickness, and, in contrast to phase 2.5, he or she can discuss the logic behind this procedure in ways that show that he or she understands it. On the other hand, this strategy is limited in that it does not recognise (a) that two features rather than one must be taken into account, and (b) that some features must be ruled out as not relevant.

A parallel to this within the houses problems may be as follows. A few students work almost entirely off the grid supplied with the problem, making up sub grids of their own and trying out combinations of items in them. Students like this may arrive at a correct solution, possibly by the long route which eliminates risk. But amongst those who work in this fashion, there are some who in the end do not present a solution at all. A student for whom this is the case typically produces a series of grids, each of which is nearly complete. Among these grids there may well be one which, if completed, would be correct. It is as if he or she cannot decide which is right, and sometimes he or she will even abandon a grid that is within one step of being correct, and develop instead another which is wrong. Such students seem unable to bring themselves finally to rule out any of the possibilities they have envisaged. The procedure they use is perhaps analogous to that of a subject in Vygotsky's exercise who cannot move from phase 3.2 to the correct solution.